

Chasing Lambda

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Summary. — Recent astronomical observations of SNIa, CMB, as well as BAO in the Sloan Digital Sky Survey, suggest that the current Universe has entered a stage of an accelerated expansion with the transition redshift at $z \simeq 0.5$. While the simplest candidates to explain this fact is cosmological constant/vacuum energy, there exists a serious problem of coincidence. In theoretical cosmology we can find many possible approaches alleviating this problem by applying new physics or other conception of dark energy. We consider state of art candidates for the description of accelerating Universe in the framework of the Bayesian model selection. We point out advantages as well as troubles of this approach. We find that the combination of four data bases gives a stringent posterior probability of the Λ CDM model which is 74%. This fact is a quantitative exemplification of a turmoil in modern cosmology over the Λ problem.

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1. – Introduction

Recent observations of SNIa stars indicate that the Universe is in an accelerating phase of expansion. If we assume that the FRW model with a source in the form of a perfect fluid well describes present evolution of the Universe, then there is only way to explain the observational fact that the Universe is accelerating, i.e. postulating some gravity source of unknown nature which violates the strong energy condition $\rho + 3p > 0$. The most natural and simplest candidate for such a type of dark energy is the vacuum energy represented by the cosmological constant Λ in the Λ CDM model. But we have still a problem with this model: predicted value of the vacuum energy density is larger than what we observe by a factor of the order 10^{120} (the so called fine tuning problem) and energy density parameters of both dark matter and dark energy are comparable at the present epoch (the so called coincidence problem). Due to that we are still looking for other alternative

TABLE I. – *The Hubble function for cosmological models with dark energy*

case	model	$\frac{H^2(z)}{H_0^2} =$
1	Λ CDM model	$= \Omega_{m,0}(1+z)^3 + (1 - \Omega_{m,0})$
2	Model with generalized Chaplygin gas	$= \Omega_{m,0}(1+z)^3 + (1 - \Omega_{m,0})[A_S + (1 - A_S)(1+z)^{3(1+\alpha)}]^{\frac{1}{1+\alpha}}$
3	Model with phantom dark energy	$= \Omega_{m,0}(1+z)^3 + (1 - \Omega_{m,0})(1+z)^{3(1+w_X)}$
4	Model with dynamical E.Q.S	$= \Omega_{m,0}(1+z)^3 + (1 - \Omega_{m,0})(1+z)^{3(w_0+w_1+1)} \exp[-\frac{3w_1 z}{1+z}]$
5	Quintessence model	$= \Omega_{m,0}(1+z)^3 + (1 - \Omega_{m,0})(1+z)^{3(1+w_0(1+z)^{-\alpha})}$

explanations of accelerating expansion phenomenon. Apart from the dark energy models we have a second group of models where the modification or generalisation of the FRW equations are postulated. We gather the most popular models belonging to the sets described before and we try to answer the question if they are better than the Λ CDM model in the light of available data sets using the Bayesian model comparison method.

2. – Cosmological models of the accelerating Universe

We consider five models belonging to the group of models with dark energy and gather them together with Hubble functions, with the assumption that the Universe is spatially flat in Table I. Here we include the Λ CDM model [1], the phantom dark energy model [2, 3], where the coefficient of the equation of state is a parameter ($w = \text{const}$) with the prior assumption that it is less than -1 , a model with dynamical equation of state [4, 5] where the coefficient of the equation of state is a linear function of a : $w(a) = w_0 + w_1(1-a)$. We also consider the quintessence model where dark energy is represented by a minimally coupled scalar field [6, 7]. We choose the power-law parameterized quintessence model, where the evolution of the dark energy density is given by $\rho_X = \rho_{X,0} a^{-3(1+\bar{w}_X(a))}$ and $\bar{w}_X(a)$ is the mean of a coefficient of the equation of state in the logarithmic scale factor $\bar{w}_X(a) = \int w_X(a) d(\ln a) / \int d(\ln a)$ with parametrization proposed by [8]: $\bar{w}_X(a) \equiv w_0 a^\alpha$. Finally we include model with the generalized Chaplygin gas [9, 10, 11], i.e. with perfect fluid which equation of state has the following form $p_X = -A/\rho_X^\alpha$, where $A > 0$ and α is a constant.

We also consider five models which belong to the the group of models with the modified or generalized FRW equation (gathered together with the Hubble functions in Table II). Here we include examples of so called brane models, which postulated that the observer is embedded on the brane in a larger space in which gravity can propagate: the Dvali-Gabadadze-Porrati model (DGP) [12], the Sahni-Shtanov brane 1 model [13, 14]. There is also the Cardassian model where the modification of the Friedmann first integral is postulated: $3H^2 = \rho + B\rho^n$, and B is a constant [15]. We also consider the bouncing model arising in the context of loop quantum gravity (the B Λ CDM model) [16, 17] and the model with energy transfer between the dark matter and dark energy sectors [18].

TABLE II. – *The Hubble function for cosmological models with modified theory of gravity.*

case	model	$\frac{H^2(z)}{H_0^2} =$
6	DGP	$= \left[\sqrt{\Omega_{m,0}(1+z)^3 + \Omega_{rc,0}} + \sqrt{\Omega_{rc,0}} \right]^2$ $\Omega_{rc,0} = \frac{(1-\Omega_{m,0})^2}{4}$
7	BACDM	$= \Omega_{m,0}(1+z)^3 - \Omega_{n,0}(1+z)^n + 1 - \Omega_{m,0} + \Omega_{n,0}$
8	interacting with Λ	$= \Omega_{m,0}(1+z)^3 + \Omega_{int,0}(1+z)^n + 1 - \Omega_{m,0} - \Omega_{int,0}$
9	Cardassian $\Omega_{r,0} = 10^{-4}$	$= \Omega_{r,0}(1+z)^4 + \Omega_{m,0}(1+z)^4 \left[\frac{1}{1+z} + (1+z)^{-4+4n} \left(\frac{1-\Omega_{r,0}-\Omega_{m,0}}{\Omega_{m,0}} \right) E(z) \right]$ $E(z) = \left(\frac{\frac{1}{1+z} + \frac{\Omega_{r,0}}{\Omega_{m,0}}}{1 + \frac{\Omega_{r,0}}{\Omega_{m,0}}} \right)^n$
10	Sahni-Shtanov brane I	$= \Omega_{m,0}(1+z)^3 + \Omega_{\sigma,0} + 2\Omega_{l,0} - 2\sqrt{\Omega_{l,0}}P(z)$ $\Omega_{\sigma,0} = 1 - \Omega_{m,0} + 2\sqrt{\Omega_{l,0}}\sqrt{1 + \Omega_{\Lambda b,0}}$ $P(z) = \sqrt{\Omega_{m,0}(1+z)^3 + \Omega_{\sigma,0} + \Omega_{l,0} + \Omega_{\Lambda b,0}}$

3. – Bayesian model comparison framework and application to cosmological models

In the Bayes theory the best model from the set under consideration is the one which has the largest value of the posterior probability [19] (the probability in the light of data D) which has the following form

$$(1) \quad P(M|D) = \frac{P(D|M)P(M)}{P(D)}.$$

$P(M)$ is the prior probability for the model considered, the value of which depends on our previous knowledge (i.e. without information coming from the data D) about the model. If we have no foundation to favour one model over another one from the set of models under consideration we usually assume the same value of the prior quantity for all models, i.e. $P(M) = 1/K$, where K is the number of models. $P(D)$ is the normalization constant. Requiring that a sum of posterior probabilities for all models from the considered set is equal to one we can obtain the form for this quantity:

$$(2) \quad \sum_{i=1}^K P(M_i|D) = 1 \longrightarrow P(D) = \sum_{i=1}^K P(D|M_i)P(M_i),$$

where i indexed the model under consideration. It should be pointed out that conclusions based on the posterior probability depend on the set of models under consideration and could change if the set is changed.

$P(D|M)$, the marginal likelihood (also called the evidence), is the most important

quantity in the Bayesian framework of model comparison and has the following form

$$(3) \quad P(D|M) = \int L(\bar{\theta}|D, M)P(\bar{\theta}|M)d\bar{\theta} \equiv E,$$

where $L(\bar{\theta}|D, M)$ is the likelihood of the model M , $\bar{\theta}$ is the vector of the model parameters and $P(\bar{\theta}|M)$ is the prior probability for the model parameters. Here we use an asymptotic approximation to the minus twice logarithm of the evidence derived by Schwarz [20], so called BIC quantity which has the following form:

$$(4) \quad \text{BIC} = -2 \ln \mathcal{L} + d \ln N,$$

where \mathcal{L} is the maximum of the likelihood function, d is the number of model parameters and N is the number of data used in the analysis. This approximation is derived with the assumption that observations ($D = \{x_i\}, i = 1, \dots, N$) used in analysis are iid (independent and identically distributed) and comes from a linear exponential distribution defined in the following way $f(x_i|\bar{\theta}) = \exp \left[\sum_{k=1}^S w_k(\bar{\theta})t_k(x_i) + b(\bar{\theta}) \right]$, where w_1, \dots, w_S, b are functions of only $\bar{\theta} \in \mathbf{R}^d$, t_1, \dots, t_S are function of only x_i and $S = d$. The prior for model parameters must be not equal to 0 in the point where the likelihood function reach maximum and the number of data must be large enough with respect to the number of model parameters (as a consequence of asymptotic approximation). This derivation was generalized by [21] where it is not required to assume any specific form for the likelihood function but is only necessary that the likelihood function satisfies some non-restrictive regularity conditions. Moreover the data do not need to be iid. This derivation requires to assume that a prior for model parameters is not equal to zero in the neighborhood of the point where the likelihood function under a given model reaches a maximum and that it is bound in the whole parameter space under consideration. The number of the data must be also large with respect to the number of model parameters.

We apply this method to compare the cosmological models described before. We make the comparison among the models with dark energy (gathered in Table I), among the models with modified theory of gravity (gathered in Table II) as well as among all models described before. We use the information coming from the SNIa, CMB, BAO and H(z) data. Let us describe those data sets.

We use $N_1 = 192$ SNIa data [22, 23, 24]. In this case the likelihood function has the following form

$$(5) \quad L_{SN} \propto \exp \left[-\frac{1}{2} \left(\sum_{i=1}^{N_1} \frac{(\mu_i^{\text{theor}} - \mu_i^{\text{obs}})^2}{\sigma_i^2} \right) \right],$$

where σ_i is known, $\mu_i^{\text{obs}} = m_i - M$ (m_i —apparent magnitude, M —absolute magnitude of SNIa), $\mu_i^{\text{theor}} = 5 \log_{10} D_{Li} + \mathcal{M}$, $\mathcal{M} = -5 \log_{10} H_0 + 25$ and $D_{Li} = H_0 d_{Li}$, where d_{Li} is the luminosity distance, which with assumption that $k = 0$ is given by $d_{Li} = (1 + z_i)c \int_0^{z_i} (dz'/H(z'))$.

We also include information coming from the CMB data. Here the likelihood function has the following form

$$(6) \quad L_R \propto \exp \left[-\frac{(R^{\text{theor}} - R^{\text{obs}})^2}{2\sigma_R^2} \right],$$

TABLE III. – *Posterior probabilities for models from Table I and for models from Table II.*

model	prior	posterior		model	prior	posterior
1	0.20	0.84		6	0.20	0.07
2	0.20	0.02		7	0.20	0.03
3	0.20	0.06		8	0.20	0.13
4	0.20	0.04		9	0.20	0.74
5	0.20	0.04		10	0.20	0.03

where R is the shift parameter, $R^{\text{theor}} = \sqrt{\Omega_{\text{m},0}} \int_0^{z_{\text{dec}}} (H_0/H(z))dz$, and $R^{\text{obs}} = 1.70 \pm 0.03$ for $z_{\text{dec}} = 1089$ [25, 26].

As third observational data we use the measurement of the baryon acoustic oscillations (BAO) from the SDSS luminous red galaxies [27]. In this case the likelihood function has the following form

$$(7) \quad L_A \propto \exp \left[-\frac{(A^{\text{theor}} - A^{\text{obs}})^2}{2\sigma_A^2} \right],$$

where $A^{\text{theor}} = \sqrt{\Omega_{\text{m},0}} (H(z_A)/H_0)^{-1/3} [(1/z_A) \int_0^{z_A} (H_0/H(z))dz]^{2/3}$ and $A^{\text{obs}} = 0.469 \pm 0.017$ for $z_A = 0.35$.

Finally we use the observational $H(z)$ data ($N_2 = 9$) from [28] (see also [29, 30] and references therein). These data are based on the differential ages (dt/dz) of the passively evolving galaxies which allow to estimate the relation $H(z) \equiv \dot{a}/a = -1/(1+z)(dz/dt)$. Here the likelihood function has the following form

$$L_H \propto \exp \left(-\frac{1}{2} \left[\sum_{i=1}^{N_2} \frac{(H(z_i) - H_i(z_i))^2}{\sigma_i^2} \right] \right),$$

where $H(z)$ is the Hubble function, H_i , z_i are observational data.

The final likelihood function is given by $L = L_{SN} L_R L_A L_H$, with the number of data points $N = 192 + 1 + 1 + 9$. We assume equal prior probabilities for all models from the set considered. Results, i.e. the values of the posterior probabilities, for the set of models with dark energy and set of models with modified theory of gravity are gathered in Table III while for the set of all models in Table IV.

We can conclude that the Λ CDM model is the best one in the set of model with dark energy as well as the best one among all considered model. If we compare only models with modified theory of gravity the best one from them is the Cardassian model.

4. – Conclusion

Our results of Bayesian method of model selection indicate that the Λ CDM is the best one. On the other hand this model has the status of an effective theory which gives us a good description, however without deep understanding. The Λ CDM model is based on the general relativity theory which is believed to be superposed by a more fundamental theory, namely quantum gravity theory. Therefore, we still search for a quantum based model of the Universe [32, 33]. So far the Λ CDM model is some effective theory of the Universe the best we have. In our opinion the Λ CDM model is expected to be emergent

TABLE IV. – *Posterior probabilities for models from Tables I and II.*

model	prior	posterior
1	0.10	0.74
2	0.10	0.02
3	0.10	0.05
4	0.10	0.04
5	0.10	0.03
6	0.10	0.01
7	0.10	0.005
8	0.10	0.01
9	0.10	0.09
10	0.10	0.005

from the future quantum gravity theory of the Universe and finding it is the challenge of the 21st century.

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